Course Change Request

New Course Proposal

Date Submitted: 10/24/24 2:41 pm

Viewing: MATH 734 : Commutative Algebra II

Last edit: 02/17/25 10:28 am

Changes proposed by: esander

In Workflow

1. **MATH Chair**
2. **SC Curriculum Committee**
3. SC Assistant Dean
4. Assoc Provost-Graduate
5. Registrar-Courses
6. Banner

Approval Path

1. 10/24/24 3:07 pm Maria Emelianenko (memelian): Approved for MATH Chair

Are you completing this form on someone else's behalf?

No

Effective Term: Fall 2025

Subject Code: MATH - Mathematics

Course Number: 734

Bundled Courses:

Is this course replacing another course? No

Equivalent Courses:

Catalog Title: Commutative Algebra II

Banner Title: Commutative Algebra II

Will section titles vary by semester? No

Credits: 3

Schedule Type: Lecture

Hours of Lecture or Seminar per week: 3

Repeatable: Not repeatable (NG) *GRADUATE ONLY*

Default Grade Mode: Graduate Regular

Recommended Prerequisite(s):
Homological algebra and/or Math 725: Algebraic geometry

Recommended Corequisite(s):
Homological algebra and/or Math 725: Algebraic geometry

Required Prerequisite(s) / Corequisite(s) (Updates only):
MATH 724: Commutative Algebra I

Registrar's Office Use Only - Required Prerequisite(s)/Corequisite(s):

And/Or	(	Course/Test Code	Min Grade/Score	Academic Level	)	Concurrency?

Registration Restrictions (Updates only):

Registrar's Office Use Only - Registration Restrictions:

Field(s) of Study:

Class(es):

Level(s):

Degree(s):

School(s):

Catalog Description:
Continuing the study of commutative Noetherian rings and their modules from Commutative Algebra I, this course is designed to cover the topics after which one could do research in commutative algebra and follow most talks at a standard conference or special session in the field. Topics include regular sequences, Cohen-Macaulay rings, Gorenstein rings, regular local rings, complete local rings, injective dimension, graded rings and modules, the Koszul complex, the canonical module, and local cohomology.

Justification:
What: creating a new course.

Why: With only the current courses in our catalog, students would be lost at a conference or special session

in commutative algebra. Commutative algebra is central to modern mathematics, and more and more students in our graduate program are interested per year. We have run preliminary versions of this course every 2 years or so, always getting a critical mass of interested and enrolled students. Courses with similar profile exist at various other graduate programs, such as the University of Kansas and the University of California at Berkeley.

Does this course cover material which crosses into another department? No

Learning Outcomes:

Descriptors for a ring's singularities such as regular and Cohen-Macaulay
Regular sequences and Cohen-Macaulay modules
Depth
Local and graded cases
Koszul complexes
Matlis duality
Complete local rings, particularly the Cohen-Structure Theorems

Will this course be scheduled as a cross-level cross listed section?

Attach Syllabus
commalg-final.pdf

Additional Attachments

Staffing:
Neil Epstein, Rebecca RG

Relationship to Existing Programs:
This will strengthen the offerings for the Math PhD students. It strengthens students' knowledge in the field of algebra.

Relationship to Existing Courses:
This is a continuation of a previous existing course, building on the previous material in order to prepare students to be ready to do research in the field.

Additional Comments:

Reviewer Comments

Key: 18787

Commutative Algebra II, sample syllabus
MATH 734

Instructor: Dr. Neil Epstein, nepstei2@gmu.edu.

Office hours: Thursday 2:30-3:30pm, Friday 10-11am, and by appointment.

Textbook: Bruns and Herzog, *Cohen-Macaulay rings* (*revised edition*), most of chapters 1–3.
I will provide supplementary materials on Hensel's Lemma and the Cohen Structure Theorem.

Required prerequisites: Algebra I (MATH 621), Commutative Algebra (MATH 724).

Recommended pre/corequisites: Homological Algebra (MATH 726) would convey a significant advantage. Algebraic Geometry (MATH 725) is also helpful for context, though far from necessary.

Course Content: This course completes the minimal basis for a student to be able to go to a conference or special session in commutative algebra and be able to follow 70% of the talks. It gives them the necessary vocabulary and parameters to understand and appreciate research in the field.

Learning Objectives

- Students will be able to manipulate regular sequences on a ring or a module.
- Students will be able to compute with Koszul complexes.
- Students will be able to construct minimal injective resolutions.
- Students will be able to compute Betti and Bass numbers.
- Students will gain facility with canonical modules.
- Students will learn the theory of Matlis duality.
- Students will touch on the basics of local cohomology.
- Students will learn the relationships between and significance of various classes of rings, especially complete, local, Cohen-Macaulay, Gorenstein, and complete rings, but also regular and complete intersection rings.

List of topics by week (section numbers from the textbook)

Week 1	homological algebra review and §1.1: regular sequences
Week 2	§§1.2 – 1.3: grade, depth, and projective dimension
Week 3	§1.4: linear algebra extended to modules. Also start §1.5
Weeks 4-5	§1.5: Graded rings and modules
Week 6	§1.6 and supplemental: The Koszul complex
Week 7	supplements: the Cohen Structure theorems and Hensel's lemma
Week 8	§2.1: Introduction to the Cohen-Macaulay property
Week 9	§2.2: Regular rings, normal rings, and Serre's conditions
Week 10	§3.1: Gorenstein rings and injective dimension over local rings
Week 11	§3.2: Injective hulls and Matlis duality
Week 12	§3.3: The canonical module
Week 13	§3.3 continued, and start §3.5
Week 14	§3.5: Local cohomology and the Local Duality Theorem

Modality: In person.

Expectations:

- Do all assigned homework problems.
- If you have any questions, ideas, or comments, speak up (but not disruptively).
- Pay attention in class and do all readings.

Grading: There will be 6-8 homework assignments given throughout the course of the semester. Hand in on the canvas site, using LaTeX formatting. Student performance on assignments is worth 100% of the grade.

Late Work: For a week after the due date of each assignment, late work will be accepted for 80% of credit. Afterwards, it will not be accepted.

Letter grades will be assigned as follows:

- 95-100%: A+. 90-94.99%: A. 85-89.99%: A-.
- 80-84.99%: B+. 75-79.99%: B. 70-74.99%: B-.
- 60-69.99%: C. 0-59.99%: F.

Attendance: Students are expected to attend and participate in all course sessions. Students are responsible for all announcements and any assignments that are announced during class.

Students with disabilities: I am happy to make arrangements with students with disabilities. These arrangements, however, must be made through the Office of Disability Services (ODS) at 993-2474. Please contact both the ODS and your instructors as soon as possible for any accommodation you might need.

Classroom behavior: If something is not clear to you, by all means, ask questions! A well-timed question can help everyone in class, even the instructor. Students are asked to be respectful and considerate of one another. If it is necessary to carry on activities that are not directly related to the material being presented in class, please leave the room and conduct these activities elsewhere. To make the most effective use of both students' and instructor's time and energy, disruptive students may be asked to leave. Students are required to comply with the directions of University officials (including faculty) who are acting within their authority to uphold a University policy. Note that any behavior that interferes with the normal operation of the teaching/learning environment is a violation of the GMU student code of conduct.

Academic standards: Standards for academic honesty protect the honest student, the reputation of GMU, and the value of the degree earned here. We should all support it both by personal honesty and by refusing to tolerate dishonesty in others. All work done must be the unique work of the individual student whose name appears on the assignment. Copying data, falsifying data, failing to give credit to referenced sources, and using generative AI to complete assignments are among violations of Academic Standards, and will be dealt with most seriously and will be addressed in accordance with university policies. The process for reporting, investigating, and adjudicating violations is outlined in the university's procedures. Consequences of violations may include academic sanctions, disciplinary actions, and other measures necessary to uphold the integrity of our academic community. Please see the university policy statement on Academic Standards, FERPA, Students with Disabilities, and Title IX compliance here: `https://stearnscenter.gmu.edu/wp-content/uploads/24-Common-GMU-Syllabus-Policies.pdf`