Course Change Request

New Course Proposal

Date Submitted: 10/24/24 2:42 pm

Viewing: MATH 726 : Homological Algebra

Last edit: 12/11/24 12:14 pm

Changes proposed by: esander

In Workflow

1. **MATH Chair**
2. **SC Curriculum Committee**
3. SC Assistant Dean
4. Assoc Provost-Graduate
5. Registrar-Courses
6. Banner

Approval Path

1. 10/24/24 3:07 pm Maria Emelianenko (memelian): Approved for MATH Chair

Are you completing this form on someone else's behalf?

No

Effective Term: Fall 2025

Subject Code: MATH - Mathematics

Course Number: 726

Bundled Courses:

Is this course replacing another course? No

Equivalent Courses:

Catalog Title: Homological Algebra

Banner Title: Homological Algebra

Will section titles vary by semester? No

Credits: 3

Schedule Type: Lecture

Hours of Lecture or Seminar per week: 3

Repeatable: Not repeatable (NG) *GRADUATE ONLY*

Default Grade Mode: Graduate Regular

Recommended Prerequisite(s):

Recommended Corequisite(s):

Required Prerequisite(s) / Corequisite(s) (Updates only):
Math 621

Registrar's Office Use Only - Required Prerequisite(s)/Corequisite(s):

And/Or	(	Course/Test Code	Min Grade/Score	Academic Level	)	Concurrency?

Registration Restrictions (Updates only):

Registrar's Office Use Only - Registration Restrictions:

Field(s) of Study:

Class(es):

Level(s):

Degree(s):

School(s):

Catalog Description:
This course is an introduction to the algebra of homology and cohomology in mathematics, which arises in various guises in algebra, geometry, combinatorics, and topology.

Students will learn about modules, chain complexes, homology, chain homotopy, Abelian categories, derived functors such as Ext and Tor, how to handle short and long exact sequences, and ways to derive information from homological invariants.

Justification:
What: Creating a new course

Why: We have run this course every two years for over a decade, albeit under the umbrella of MATH 629. The content is important to algebra, geometry, combinatorics, and topology. It is standard at many math graduate programs, e.g. the University of Nebraska and Harvard University.

Does this course cover material which crosses into another department? No

Learning Outcomes:

Chain complexes
Exact sequences
Global dimensions
Resolutions
Chain homotopies
Ext and Tor
Abelian categories

Will this course be scheduled as a cross-level cross listed section? No

Attach Syllabus
homological.pdf

Additional Attachments

Staffing:
Neil Epstein, Rebecca R.G., Rebecca Goldin, Thomas Wanner

Relationship to Existing Programs:
This will strengthen the offerings for the Math PhD students. It strengthens students' knowledge in the field of algebra.

Relationship to Existing Courses:
This course fills a gap in the algebra/topology offerings.

Additional Comments:

Reviewer Comments

Key: 18788

Homological Algebra, sample syllabus
MATH 726

Instructor: Dr. Neil Epstein, nepstei2@gmu.edu.

Office hours: Thursday 2:30-3:30pm, Friday 10-11am, and by appointment.

Textbooks:

- Vermani, *An Elementary Approach to Homological Algebra* (first half of course), chapters 1–7
- Weibel, *An Introduction to Homological Algebra* (second half of course), chapters 2–4
- We also use a section from Mac Lane, *Categories for the Working Mathematician*

Prerequisites: Algebra I (MATH 621).

Course Content This course is an introduction to the theory and practice of homological algebra. The basic objects will be modules over a ring, but connections will be made to other things to which homological techniques apply. Topics include modules, chain complexes, homology, chain homotopy, Abelian categories, exact sequences, homological invariants, and derived functors.

Learning objectives

- Students will review the basics of modules over a ring.
- Students will learn about the special classes of modules relevant to homological algebra, especially the projective, injective, free, and flat modules.
- Students will be able to compute with chain complexes, homology, and chain homotopy.
- Students will develop intuition (and computational and proof techniques) for passing from categories of modules to more general Abelian categories
- Students will gain facility with derived functors such as Ext and Tor.
- Students will be able to handle short and long exact sequences.
- Students will be able to to analyze and extract information from homological invariants.
- Students will be able to communicate their knowledge about this subject orally and in writing,

List of topics by week

Week 1	Introduction to homology; Introduction to modules
Week 2	Exact sequences; the Hom functor
Week 3	Tensor products; Category theory
Week 4	Abelian categories; functors; exactness; bifunctors
Week 5	Projective, flat, injective, divisible, and torsion-free modules; Hom-tensor adjointness
Week 6	"Enough" injective modules; The Snake Lemma; the Long Exact Sequence theorem; Chain homotopy; quasi-isomorphisms and homotopy equivalences
Week 7	Complexes in abelian categories; Projective resolutions; The Comparison Theorem; Injective resolutions; Opposite categories
Week 8	Derived functors; the Horseshoe lemma; δ-functors; Tor and Ext; dimension shifting
Week 9	flexibility in computing Tor and Ext; T-acyclic objects and resolutions; the Hom-evaluation map
Week 10	Homological (projective, injective, and flat) dimensions; the Global Dimension Theorem; the Tor-dimension theorem
Week 11	Rings with small homological dimensions
Week 12	Pullbacks and pushouts; Membership as a proxy for elementhood in abelian categories
Week 13	the Snake Lemma revisited; Ext, extensions, and the Baer sum
Week 14	application of global dimension to commutative Noetherian local rings; group homology and cohomology

Modality: In person.

Expectations:

- I hope this material is fun and interesting! It should be useful for various further studies in algebra, geometry, and topology.
- Do all assigned homework problems.
- If you have any questions, ideas, or comments, speak up (but not disruptively).
- Pay attention in class and do all readings.

Grading: There will be 6-8 homework assignments given throughout the course of the semester. Hand in on the canvas site, using LaTeX formatting. Student performance on assignments is worth 100% of the grade.

Late Work: For a week after the due date of each assignment, late work will be accepted for 80% of credit. Afterwards, it will not be accepted.

Letter grades will be assigned as follows:

- 95-100%: A+. 90-94.99%: A. 85-89.99%: A-.
- 80-84.99%: B+. 75-79.99%: B. 70-74.99%: B-.
- 60-69.99%: C. 0-59.99%: F.

Attendance: Students are expected to attend and participate in all course sessions. Students are responsible for all announcements and any assignments that are announced during class.

Students with disabilities: I am happy to make arrangements with students with disabilities. These arrangements, however, must be made through the Office of Disability Services (ODS) at 993-2474. Please contact both the ODS and your instructors as soon as possible for any accommodation you might need.

Classroom behavior: If something is not clear to you, by all means, ask questions! A well-timed question can help everyone in class, even the instructor. Students are asked to be respectful and considerate of one another. If it is necessary to carry on activities that are not directly related to the material being presented in class, please leave the room and conduct these activities elsewhere. To make the most effective use of both students' and instructor's time and energy, disruptive students may be asked to leave. Students are required to comply with the directions of University officials (including faculty) who are acting within their authority to uphold a University policy. Note that any behavior that interferes with the normal operation of the teaching/learning environment is a violation of the GMU student code of conduct.

Academic standards: Standards for academic honesty protect the honest student, the reputation of GMU, and the value of the degree earned here. We should all support it both by personal honesty and by refusing to tolerate dishonesty in others. All work done must be the unique work of the individual student whose name appears on the assignment. Copying data, falsifying data, failing to give credit to referenced sources, and using generative AI to complete assignments are among violations of Academic Standards, and will be dealt with most seriously and will be addressed in accordance with university policies. The process for reporting, investigating, and adjudicating violations is outlined in the university's procedures. Consequences of violations may include academic sanctions, disciplinary actions, and other measures necessary to uphold the integrity of our academic community. Please see the university policy statement on Academic Standards, FERPA, Students with Disabilities, and Title IX compliance here: `https://stearnscenter.gmu.edu/wp-content/uploads/24-Common-GMU-Syllabus-Policies.pdf`