Course Change Request

New Course Proposal

Date Submitted: 10/24/24 2:41 pm

Viewing: MATH 734 : Commutative Algebra II

Last edit: 11/06/24 9:11 am

Changes proposed by: esander

Are you completing this form on someone else's behalf?

No

In Workflow

1. **MATH Chair**
2. **SC Curriculum Committee**
3. SC Assistant Dean
4. Assoc Provost-Graduate
5. Registrar-Courses
6. Banner

Approval Path

1. 10/24/24 3:07 pm Maria Emelianenko (memelian): Approved for MATH Chair

Effective Term: Fall 2025

Subject Code: MATH - Mathematics

Course Number: 734

Bundled Courses:

Is this course replacing another course? No

Equivalent Courses:

Catalog Title: Commutative Algebra II

Banner Title: Commutative Algebra II

Will section titles vary by semester? No

Credits: 3

Schedule Type: Lecture

Hours of Lecture or Seminar per week: 3

Repeatable:

May be only taken once for credit, limited to 3 attempts (N3)

Max Allowable Credits: 9

Default Grade Mode: Graduate Regular

Recommended Prerequisite(s):
Homological algebra and/or Math 725: Algebraic geometry

Recommended Corequisite(s):
Homological algebra and/or Math 725: Algebraic geometry

Required Prerequisite(s) / Corequisite(s) (Updates only):
MATH 724: Commutative Algebra I

Registrar's Office Use Only - Required Prerequisite(s)/Corequisite(s):

And/Or	(	Course/Test Code	Min Grade/Score	Academic Level	)	Concurrency?

Registration Restrictions (Updates only):

Registrar's Office Use Only - Registration Restrictions:

Field(s) of Study:

Class(es):

Level(s):

Degree(s):

School(s):

Catalog Description:
Continuing the study of commutative Noetherian rings and their modules from Commutative Algebra I, this course is designed to cover the topics after which one could do research in commutative algebra and follow most talks at a standard conference or special session in the field. Topics include regular sequences, Cohen-Macaulay rings, Gorenstein rings, regular local rings, complete local rings, injective dimension, graded rings and modules, the Koszul complex, the canonical module, and local cohomology.

Justification:

What: creating a new course.

Why: With only the current courses in our catalog, students would be lost at a conference or special session in commutative algebra. Commutative algebra is central to modern mathematics, and more and more students in our graduate program are interested per year. We have run preliminary versions of this course every 2 years or so, always getting a critical mass of interested and enrolled students. Courses with similar profile exist at various other graduate programs, such as the University of Kansas and the University of California at Berkeley.

Does this course cover material which crosses into another department? No

Learning Outcomes:

Descriptors for a ring's singularities such as regular and Cohen-Macaulay
Regular sequences and Cohen-Macaulay modules
Depth
Local and graded cases
Koszul complexes
Matlis duality
Complete local rings, particularly the Cohen-Structure Theorems

Will this course be scheduled as a cross-level cross listed section?

Attach Syllabus
commalgii.pdf

Additional Attachments

Staffing:
Neil Epstein, Rebecca RG

Relationship to Existing Programs:
This will strengthen the offerings for the Math PhD students. It strengthens students' knowledge in the field of algebra.

Relationship to Existing Courses:
This is a continuation of a previous existing course, building on the previous material in order to prepare students to be ready to do research in the field.

Additional Comments:

Reviewer Comments

Key: 18787

Commutative Algebra II, sample syllabus
MATH 734

Potential instructors: Neil Epstein, Rebecca R.G.

Frequency: It would be ideal to run this course every two or (at least) every three years.

Textbooks: Bruns and Herzog, *Cohen-Macaulay rings* (*revised edition*)

Required prerequisites: Algebra I (MATH 621), Commutative Algebra (MATH 724).

Recommended pre/corequisites: Homological algebra (course number to be determined) would convey a significant advantage. Algebraic Geometry (MATH 725) is also helpful for context, though far from necessary.

Course Content: Most of Chapters 1-3 of Bruns and Herzog, along with supplementary material on Hensel's lemma and the Cohen structure theorem.

This course completes the minimal basis for a student to be able to go to a conference or special session in commutative algebra and be able to follow 70% of the talks. It gives them the necessary vocabulary and parameters to understand and appreciate research in the field. Students will learn about regular sequences, Koszul complexes, minimal injective resolutions, Betti and Bass numbers, canonical modules, Matlis duality, and local cohomology. They will learn the relationships between and significance of various classes of rings, especially Cohen-Macaulay, Gorenstein, and complete rings, but also regular and complete intersection rings.

Expectations:

- Do all assigned homework problems.
- If you have any questions, ideas, or comments, speak up (but not disruptively).
- Pay attention in class and do all readings.

Grading: We have run this course a couple different ways. As a lecture-based course, I have assigned 6-8 homework assignments, graded for completion. The other style has been for students to take turns presenting the material to the other students, graded on quality of presentation.

Relevant George Mason
Official University Policies

The following policies apply to all courses at George Mason University:

1. Attendance and class participation is a portion of your grade. You will be graded on attending and actively participating in class discussions.
2. In general, late assignments are not accepted except in exceptional circumstances. Likewise, makeup for missed exams is only with clear documentation of a valid reason for missing the exam.
3. Feel free to send an email with your preferred name and gender pronoun. Note that Mason provides online methods for students to change their name and pronouns on Mason records.
4. This classroom fosters inclusivity in keeping with George Mason's core values.
5. None of the assignments will involve laptops or other electronic devices.
6. The Common Policies Addendum (via online link, PDF or document text), with policies about Academic Standards, Accommodations for Students with Disabilities, FERPA, and Title IX.
7. You are responsible for the accuracy of your own schedule. Check Patriot Web regularly to verify that you are registered for the classes that you think you are. A student who is not registered may not continue to attend class. Faculty are not permitted to grade work of students who do not appear on the official class roster.
8. You are responsible for knowing the last days to drop and add this class.
9. Once the add and drop deadlines have passed, instructors do not have the authority to approve any requests from students to add or drop/withdraw late. It is NOT permissible to drop the class and leave it at that. It needs approval. Late adds (up until the last day of classes) must be reviewed and approved by the department chair of the course being offered. They will be approved only in the case of a documented university error (such as a problem with Financial Aid being processed). All student requests for withdrawals and retroactive adds (after the last day of classes) must be reviewed by the student's academic dean. In the case of students whose major is in COS, this is the office of Undergraduate Academic Affairs in Enterprise.
10. Instructors are required to give the final exam at the time and place published in the Schedule of Classes, as set by the Registrar. It cannot change be changed. You need to plan vacation (make plane reservations, etc.) around these published dates.
11. Once final grades have been recorded, instructors cannot accept any work to change that course grade. Grade changes can only be approved when they are due to a calculation or recording error on the part of the instructor.
12. An IN (incomplete) grade is a very special grade that can only be applied for in writing. It can only be given in cases in which a student is passing a course and has a very limited amount of work left to complete the course.
13. Federal law (a law known as FERPA) requires the protection of privacy of student information. Therefore, no instructor on campus can speak about a student's record with anyone other than the student. The record includes how a student is doing in a course, whether a student has attended class, information about grades, whether a paper has been turned in. Anything. This prohibition includes parents, siblings, and spouses, anyone.

List of topics by week

Week 1	homological algebra review and §1.1: regular sequences
Week 2	§§1.2 – 1.3: grade, depth, and projective dimension
Week 3	§1.4: linear algebra extended to modules. Also start §1.5
Weeks 4-5	§1.5: Graded rings and modules
Week 6	§1.6 and supplemental: The Koszul complex
Week 7	supplements: the Cohen Structure theorems and Hensel's lemma
Week 8	§2.1: Introduction to the Cohen-Macaulay property
Week 9	§2.2: Regular rings, normal rings, and Serre's conditions
Week 10	§3.1: Gorenstein rings and injective dimension over local rings
Week 11	§3.2: Injective hulls and Matlis duality
Week 12	§3.3: The canonical module
Week 13	§3.3 continued, and start §3.5
Week 14	§3.5: Local cohomology and the Local Duality Theorem