Course Change Request

New Course Proposal

Date Submitted: 10/24/24 2:42 pm

Viewing: MATH 726 : Homological Algebra

Last edit: 12/02/24 10:06 am

Changes proposed by: esander

In Workflow

1. **MATH Chair**
2. **SC Curriculum Committee**
3. SC Assistant Dean
4. Assoc Provost-Graduate
5. Registrar-Courses
6. Banner

Approval Path

1. 10/24/24 3:07 pm Maria Emelianenko (memelian): Approved for MATH Chair

Are you completing this form on someone else's behalf?

No

Effective Term: Fall 2025

Subject Code: MATH - Mathematics

Course Number: 726

Bundled Courses:

Is this course replacing another course? No

Equivalent Courses:

Catalog Title: Homological Algebra

Banner Title: Homological Algebra

Will section titles vary by semester? No

Credits: 3

Schedule Type: Lecture

Hours of Lecture or Seminar per week: 3

Repeatable:

May be only taken once for credit, limited to 3 attempts (N3)

Max Allowable Credits: 9

Default Grade Mode: Graduate Regular

Recommended Prerequisite(s):

Recommended Corequisite(s):

Required Prerequisite(s) / Corequisite(s) (Updates only):

Math 621

Registrar's Office Use Only - Required Prerequisite(s)/Corequisite(s):

And/Or	(	Course/Test Code	Min Grade/Score	Academic Level	)	Concurrency?

Registration Restrictions (Updates only):

Registrar's Office Use Only - Registration Restrictions:

Field(s) of Study:

Class(es):

Level(s):

Degree(s):

School(s):

Catalog Description:

This course is an introduction to the algebra of homology and cohomology in mathematics, which arises in various guises in algebra, geometry, combinatorics, and topology.

Students will learn about modules, chain complexes, homology, chain homotopy, Abelian categories, derived functors such as Ext and Tor, how to handle short and long exact sequences, and ways to derive information from homological invariants.

Justification:

What: Creating a new course

Why: We have run this course every two years for over a decade, albeit under the umbrella of MATH 629. The content is important to algebra, geometry, combinatorics, and topology. It is standard at many math graduate programs, e.g. the University of Nebraska and Harvard University.

Does this course cover material which crosses into another department? No

Learning Outcomes:

Chain complexes
Exact sequences
Global dimensions
Resolutions
Chain homotopies
Ext and Tor
Abelian categories

Will this course be scheduled as a cross-level cross listed section? No

Attach Syllabus

homological.pdf

Additional Attachments

Staffing:

Neil Epstein, Rebecca R.G., Rebecca Goldin, Thomas Wanner

Relationship to Existing Programs:

This will strengthen the offerings for the Math PhD students. It strengthens students' knowledge in the field of algebra.

Relationship to Existing Courses:

This course fills a gap in the algebra/topology offerings.

Additional Comments:

Reviewer Comments

Key: 18788

Homological Algebra, sample syllabus
MATH 726

Potential instructors: Neil Epstein, Rebecca R.G., Rebecca Goldin, or Thomas Wanner.

Frequency: This course has already been on rotation to be run every two years for over a decade. We should continue this.

Textbooks: As currently run, here's what I do.

- Vermani, *An Elementary Approach to Homological Algebra* (first half of course)
- Weibel, *An Introduction to Homological Algebra* (second half of course)
- We also use a section from Mac Lane: *Categories for the Working Mathematician*

A previous instructor (Jay Shapiro, emeritus) previously used Rotman's *An introduction to homological algebra*

Prerequisites: Algebra I (MATH 621).

Course Content:: Chapters 1-7 of Vermani, the Diagram Lemmas section from Mac Lane, and parts of chapters 2,3,4 of Weibel.

Students will learn about modules, chain complexes, homology, chain homotopy, Abelian categories, derived functors such as Ext and Tor, how to handle short and long exact sequences, and ways to derive information from homological invariants.

Expectations:

- Do all assigned homework problems.
- If you have any questions, ideas, or comments, speak up (but not disruptively).
- Pay attention in class and do all readings.

Grading: I give 6-8 homework assignments throughout the course of the semester. I grade mostly according to completeness.

Relevant George Mason
Official University Policies

The following policies apply to all courses at George Mason University:

1. Attendance and class participation is a portion of your grade. You will be graded on attending and actively participating in class discussions.
2. In general, late assignments are not accepted except in exceptional circumstances. Likewise, makeup for missed exams is only with clear documentation of a valid reason for missing the exam.
3. Feel free to send an email with your preferred name and gender pronoun. Note that Mason provides online methods for students to change their name and pronouns on Mason records.
4. This classroom fosters inclusivity in keeping with George Mason's core values.
5. None of the assignments will involve laptops or other electronic devices.
6. The Common Policies Addendum (via online link, PDF or document text), with policies about Academic Standards, Accommodations for Students with Disabilities, FERPA, and Title IX.
7. You are responsible for the accuracy of your own schedule. Check Patriot Web regularly to verify that you are registered for the classes that you think you are. A student who is not registered may not continue to attend class. Faculty are not permitted to grade work of students who do not appear on the official class roster.
8. You are responsible for knowing the last days to drop and add this class.
9. Once the add and drop deadlines have passed, instructors do not have the authority to approve any requests from students to add or drop/withdraw late. It is NOT permissible to drop the class and leave it at that. It needs approval. Late adds (up until the last day of classes) must be reviewed and approved by the department chair of the course being offered. They will be approved only in the case of a documented university error (such as a problem with Financial Aid being processed). All student requests for withdrawals and retroactive adds (after the last day of classes) must be reviewed by the student's academic dean. In the case of students whose major is in COS, this is the office of Undergraduate Academic Affairs in Enterprise.
10. Instructors are required to give the final exam at the time and place published in the Schedule of Classes, as set by the Registrar. It cannot change be changed. You need to plan vacation (make plane reservations, etc.) around these published dates.
11. Once final grades have been recorded, instructors cannot accept any work to change that course grade. Grade changes can only be approved when they are due to a calculation or recording error on the part of the instructor.
12. An IN (incomplete) grade is a very special grade that can only be applied for in writing. It can only be given in cases in which a student is passing a course and has a very limited amount of work left to complete the course.
13. Federal law (a law known as FERPA) requires the protection of privacy of student information. Therefore, no instructor on campus can speak about a student's record with anyone other than the student. The record includes how a student is doing in a course, whether a student has attended class, information about grades, whether a paper has been turned in. Anything. This prohibition includes parents, siblings, and spouses, anyone.

List of topics by week

Week 1	Introduction to homology; Introduction to modules
Week 2	Exact sequences; the Hom functor
Week 3	Tensor products; Category theory
Week 4	Abelian categories; functors; exactness; bifunctors
Week 5	Projective, flat, injective, divisible, and torsion-free modules; Hom-tensor adjointness
Week 6	"Enough" injective modules; The Snake Lemma; the Long Exact Sequence theorem; Chain homotopy; quasi-isomorphisms and homotopy equivalences
Week 7	Complexes in abelian categories; Projective resolutions; The Comparison Theorem; Injective resolutions; Opposite categories
Week 8	Derived functors; the Horseshoe lemma; δ-functors; Tor and Ext; dimension shifting
Week 9	flexibility in computing Tor and Ext; T-acyclic objects and resolutions; the Hom-evaluation map
Week 10	Homological (projective, injective, and flat) dimensions; the Global Dimension Theorem; the Tor-dimension theorem
Week 11	Rings with small homological dimensions
Week 12	Pullbacks and pushouts; Membership as a proxy for elementhood in abelian categories
Week 13	the Snake Lemma revisited; Ext, extensions, and the Baer sum
Week 14	application of global dimension to commutative Noetherian local rings; group homology and cohomology